



Systematic Analysis of Learning Models to Improve Students' Mathematical Representation Skills

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ABSTRACT

This study aims to systematically analyze the learning model used in improving students' mathematical representation skills. The research approach used is Systematic Literature Review (SLR) with reference to the PRISMA flow, which involves the process of identification, selection, and synthesis of Scopus indexed articles published in the range of 2020–2025. From the results of the screening of data sources from Scopus, ScienceDirect, SpringerLink, Taylor & Francis, Wiley, and MDPI, five core articles were obtained that met the criteria of relevance and quality. The analysis was carried out through thematic *coding techniques* to identify the focus of learning interventions, representational strategies, and the achievement of students' mathematical representation skills. The results of the analysis showed that learning models such as Problem-Based Learning, Realistic Mathematics Education, Brain-Based Learning-based TANDUR, Think Talk-Write, and GeoGebra-assisted Discovery Learning consistently improved students' verbal, symbolic, and visual representations. Improvement occurs when learning provides meaningful context, structured representational scaffolding, and visual-interactive manipulation as a medium for concept exploration. In conclusion, the effectiveness of improving mathematical representation skills is not only determined by the chosen learning model, but also by the quality of pedagogical design and the consistency of teachers in systematically integrating representational strategies in the learning process.

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Introduction

Mathematical representation skills are one of the important cognitive competencies that determine students' success in understanding concepts, solving problems, and communicating mathematical ideas sequentially. Representation is not only related to the ability to write down symbols and notations, but also includes the ability to translate mathematical information into various forms such as graphs, tables, figures, equations, diagrams, visual models, and verbal descriptions (Pedersen et al., 2021). However, the real conditions in the field show that most students still have difficulty in connecting these various forms of representation. Students often rely on only one form of representation—usually symbolic—without being able to relate it to other, more intuitive representations such as diagrams or graphs (Bicer, 2021). As a result, students tend to experience misconceptions, procedural errors, and low problem-solving skills.

This problem is exacerbated by the dominance of conventional learning in many schools, especially those that still emphasize routine and procedural practice rather than conceptual understanding. Teachers tend to deliver material mechanically and are oriented towards the final answer, not the thought process (Septian et al., 2020). In this condition, students' mathematical representation skills are not optimally developed because students are not given space to explore the relationships between forms of representation or construct understanding independently. In addition, the low integration of educational technology and

visual-interactive learning resources is also an obstacle in strengthening representation skills (Rabi et al., 2021).

Ideally, math learning should provide students with the opportunity to organize and construct meaning through interaction with various forms of representation. An effective learning approach should lead students to interpret, convert, and relate verbal, symbolic, graphic, and visual representations so that a holistic conceptual understanding is formed (Li et al., 2022). The concept of *multiple representation learning environment* (MRLE) in this case is the foundation that representation abilities do not develop automatically, but must be designed through learning models that support exploration, manipulation, and representational reflection (Arefaine et al., 2022). Ideal learning also requires teachers to design open-ended assignments, provide representational scaffolding, and provide collaborative interaction time so that students practice building meaningful relationships between representations (Ceuppens et al., 2018).

However, there are *important gaps* that still often occur in learning practices. Various studies show that the learning model used by teachers has not been systematically designed to develop mathematical representation skills (Mpuangnan et al., 2024). The learning design often stops at the manipulation of only one form of representation and does not lead students to the process of translation between representations. In addition, although a number of studies have shown the effectiveness of *Problem-Based Learning (PBL)*, *GeoGebra-Assisted Learning*, *Learning Cycle 5E*, *STEM-PjBL*, and *Multiple Representation Approach* on improving mathematical representation, the results still vary depending on the assignment design, material context, and teacher mentoring strategies (Mayasari et al., 2024; Dianningrum & Sufian, 2025). This indicates that learning practices are still not standardized and there is no comprehensive synthesis that explains the characteristics of what learning models are most effective for improving mathematical representation skills.

Based on these conditions, a systematic analysis of the learning models that have been used in an effort to improve students' mathematical representation skills is needed. This systematic analysis is important to identify: (1) the most widely used types of learning models and the context in which they are applied, (2) learning implementation strategies that have been proven to be effective in improving mathematical representation, (3) the forms of representational support provided by teachers during the learning process, and (4) the learning factors that affect the success of improving students' mathematical representation skills. In other words, this study not only aims to collect references, but also to conduct conceptual and methodological synthesis of various studies that have been carried out (Pierson, 2023).

The novelty of this study lies in the focus of a synthesis that not only examines the effectiveness of each learning model separately, but also analyzes the core characteristics of learning designs that contribute directly to the strengthening of mathematical representation skills. This research not only explains *what* the learning model is, but also *how* it works, *why* it is effective, and in *what* context it is best applied. Thus, this research produces a conceptual map that provides a direction for the development of a learning model based on mathematical representations that is more structured, adaptive, and relevant for implementation in the classroom.

Method

This study uses the Systematic Literature Review (SLR) method with reference to the PRISMA flow. The research data sources are from the Scopus, ScienceDirect, SpringerLink, Taylor & Francis, Wiley, and MDPI databases with a publication range of 2020–2025. Included articles must meet the following criteria: (1) Scopus indexed, (2) discuss learning models that explicitly target improving students' mathematical representation skills, (3) be in the form of empirical studies (experimental, quasi-experimental, or mixed methods), and (4) be available in *full text*. Articles that do not contain empirical data or do not measure mathematical

representations are excluded. From the screening process, 5 articles were obtained as the main source of analysis.

Data analysis was carried out through thematic coding to identify learning model patterns, instructional strategies, forms of representational support, and the results of mathematical representation skills. Each article was examined on aspects of learning design, indicators of representation assessment, and the level of effectiveness. The results of the analysis are synthesized comparatively to find the tendency of the most effective learning model and the conditions for its application. Validity is supported through consistency of selection criteria and *cross-check* between findings.

Results and Discussion

After the selection process and literature review were carried out, 5 articles were obtained that met the criteria and became the basis for analysis in this study. The articles cover a variety of learning models designed to improve students' mathematical representation skills, such as Problem Based Learning, Realistic Mathematics Education, Brain-Based Learning, Technology-Assisted Discovery Learning, as well as collaborative strategies such as Think-Talk-Write and contextual learning. Each article was analyzed based on the research design, characteristics of the application of the model, and indicators of measurement of mathematical representations which included visual, symbolic, and verbal representations. The emphasis on analysis is directed at how the learning model facilitates students' thinking processes in converting mathematical information into appropriate representations. The results of this synthesis are then presented to show the patterns of effectiveness, the most prominent approach tendencies, and research gaps that are still open in the development of mathematics learning.

Table 1. Article review results

Author & Year	Article Title	Research Methods	Key Findings
Hasibuan, A., Saragih, S., & Amry, Z. (2021)	<i>Development of PBL Model to Improve Mathematical Representation Ability and Self-Efficacy of Students</i>	Research and Development (PBL learning model developed and tested in experimental classes)	The PBL model developed significantly improved the mathematical representation capabilities compared to the control class. An increase also occurred in the aspect of students' confidence in learning mathematics.
Hidayati, L., & Faradillah, S. (2022)	<i>The Effect of Realistic Mathematics Education on Students' Representation Ability</i>	Quasi-Experimental Design (pretest-posttest control group)	The consistent application of RME makes students able to visualize problems into mathematical models better than conventional approaches.
Thahir, A., & Abdullah, M. (2023)	<i>TANDUR Learning Model Based on Brain-Based Learning to Enhance Students' Mathematical Representation Skills</i>	Quasi-Experimental (experimental and control class)	The TANDUR model significantly improves symbolic and verbal representation, especially at the stage of exploration and presentation of problems.
Yuniarti, D., & Rahman, H. (2020)	<i>The Influence of Think-Talk-Write Strategy on Mathematical</i>	Experimental Design	The Think-Talk-Write strategy encourages a gradual process of processing information, so that

Author & Year	Article Title	Research Methods	Key Findings
Putri, N., & Herman, T. (2021)	<i>Representation Ability of Junior High School Students</i>	Classroom Action Research (CAR)	students are able to transfer verbal ideas to mathematical writing more accurately.
	<i>Using GeoGebra-Assisted Discovery Learning to Improve Students' Mathematical Representation Skill</i>		The use of GeoGebra in Discovery Learning enhances graphic and visual representations significantly, as students can manipulate objects and validate forms of representation independently.

The results of this study show that students' mathematical representation skills are consistently improved through the application of learning models that provide space for the construction of meaning, exploration of representations, and targeted mathematical communication. The results of the research by Hasibuan, Saragih, and Amry (2021) which emphasized the success of PBL in encouraging the translation of mathematical representations are in line with the findings of Amanah, Suryadi, and Herman (2021) and Dewi and Rahmat (2023) who also noted that PBL is effective when students are given open problems that demand the presentation of ideas in various forms of representation. However, on the other hand, a study by Rahayu and Setiawan (2022) shows that PBL can be less effective in students with low basic abilities if teachers do not provide gradual scaffolding. This shows that the effectiveness of PBL does not lie solely in the model, but in the quality of pedagogical assistance.

In addition, the findings of Hidayati and Faradillah (2022) regarding the success of Realistic Mathematics Education (RME) in facilitating the formation of representations through real context are consistent with the studies of Putra and Nugroho (2020) and Lee and Rahman (2024), which found that context-based learning improves the fluency of visual and symbolic representations. However, several other studies such as Tanuwijaya (2023) criticize that RMEs tend to lack high-level abstract representations if the context used is too tied to the students' concrete experiences. This shows that the selection of contexts in the RME should be designed gradually from realistic to ideal-mathematical.

Furthermore, the effectiveness of the TANDUR model reported by Thahir and Abdullah (2023) in strengthening symbolic and verbal representations through a neurological approach based on memory reinforcement is strengthened by the findings of Wibowo and Amin (2022), which affirm the importance of *repetition structure* in the internalization of mathematical concepts. However, this cognitive-based model underemphasizes aspects of discourse and social construction, which some studies such as Yuanita and Sari (2021) have stated to be important factors in the formation of sustainable representation.

Yuniarti and Rahman's (2020) research on the Think-Talk-Write strategy shows that dialogue and verbal articulation before mathematical writing are key to the consolidation of understanding a finding that is reinforced by the research of Arifin and Mulyono (2023), who state that verbal mathematical communication plays an important role in overcoming representational misconceptions. However, the study of Sulastri (2021) highlights that the effectiveness of this strategy is greatly influenced by the heterogeneity of students' communication skills in the classroom.

Meanwhile, the use of GeoGebra in the research of Putri and Herman (2021) is in line with many other studies in the reference list such as Zhou and Lin (2022) and Nurhayati and Wijaya (2023), which show that visual-interactive manipulation is very effective in improving graphical representation and understanding of variable relationships. However, Junaedi's (2024) study emphasizes that the success of technology integration is highly dependent on teacher readiness, so this model has higher demands on digital pedagogical competence than other learning models.

Overall, this comparison shows that no single learning model is the absolute best, but rather that effectiveness increases when it integrates several important elements simultaneously: meaningful contextualization, explicit representation scaffolding, visual-interactive manipulation, and structured mathematical communication. Thus, the success of improving mathematical representation skills is largely determined by how teachers combine these approaches in learning design.

Conclusion

The results of this study show that the improvement of students' mathematical representation skills does not depend only on the selection of a particular learning model, but on how the model is implemented through a series of pedagogical strategies that support the construction of meaning. The integration of meaningful contexts as in RME and PBL, the explicit presentation of representational translations, the use of visual-interactive manipulative media such as GeoGebra, and the facilitation of structured mathematical communication through the Think-Talk-Write strategy have proven to be a consistent combination of improving students' verbal, symbolic, and visual representations. These findings confirm that mathematical representations develop when students are encouraged to actively construct, describe, and verify mathematical ideas in various forms of presentation.

Thus, the success of learning interventions is not determined by the learning model itself, but by the quality of learning design and the consistency of the teacher's application in providing systematic representational scaffolding. The practical implications of this study are the need for teacher training that focuses on the design of layered representational tasks, standardized representation assessments, and the integration of interactive technology as a tool for concept exploration. Follow-up research is recommended to use long-term test designs with larger sample sizes to assess the robustness of mathematical representations and their transfer to different materials.

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